Airborne electromagnetics: dealing with the aircraft speed

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SUMMARY

It is no secret that the solution of Maxwell's equations depends on the coordinate system. But in current studies, the dependence of the solution on both the speed of the transmitter and the speed of the receiver is usually not discussed.

In this article, I present an analysis of such an effect on the readings of an alternating magnetic field receiver and on the secondary field. I have found that the effect of the receiver's motion is critical. I have proposed a compensation method now implemented in some systems, after which the measurements of a moving receiver can be considered as signals of an equivalent stationary receiver at the current position.

It is also shown that the field distortions proportional to the aircraft speed are related to the flight altitude and the electrical conductivity of the medium. I analyzed data from the EQUATOR airborne electromagnetic system obtained over the sea surface. It is shown that the influence of speed is much less than the influence of restrictions on the environment model, which are imposed during the inversion of airborne electromagnetic data.

Key words: airborne electromagnetic survey; aircraft speed, EQUATOR.

INTRODUCTION

In my experience the question about the speed influence appeared twice. The first time was related to the very first flight of the AEM time-domain system EQUATOR in 2010 (Moilanen et al., 2013). While we were trying to analyze AEM data in frequency domain, it was necessary to get high quality measurements on-time, during the pulse of the primary field. After all possible compensations there was still a valuable signal which was obviously related to the receiver angular motion. A solution was founded and it is presented in this paper.

The second time was quite recently, during the Neretva river AEM surveying in 2021, again with EQUATOR system. I found that when approaching the Adriatic sea coastline from the water area, the residuals of the obtained solution for the 1D inversion increase noticeably. Possible causes needed to be explored. Since the electrical conductivity of the sea water is very high, a hypothesis arose about the influence of the speed of the AEM system. Indeed, according to Landau et al. (1984), Maxwell's equations depend on the reference frame in which they are written:

$$\nabla\times(\mathbf{E}-\mathbf{B}\times\mathbf{v}) = -\frac{\partial\mathbf{B}}{\partial t},$$
$$\nabla\times(\mathbf{B}+\mu_0\varepsilon_0\mathbf{E}\times\mathbf{v}) = \sigma\mu_0\mathbf{E}-\mu_0\rho\mathbf{v}+\mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}+\mu_0\mathbf{J}^{st}. \quad (1)$$

Here σ is the conductivity, ε_0 is the permittivity, μ_0 is the magnetic permeability, ρ is the charge density, $\mathbf{v}$ is the coordinate system velocity vector, $\mathbf{J}^{st}$ denotes the external currents density, $\mathbf{E}$ is the electric field strength vector, and $\mathbf{B}$ is the magnetic field induction vector.

However, when considering publications related to AEM, the only aspect in which the influence of speed is considered is the possibility of missing a small target. This applies to the basic works of the last quarter of the 20^{th} century, when the AEM method had already become established all over the world (Won and Smits , 1987, Becker and Cheng, 1988). Little has changed in the 21^{st} century. In works considered to be a general overview of methods and tasks of AEM, the direct influence of speed on the measurements is not considered (Christiansen et al., 2006, Macnae, 2007, Kamenetsky et al., 2010, Legault, 2015, Moilanen, 2022).

Thus, the problem turns out to be unexplored. Further, I present the studying results of the influence of the movement of the receiver and transmitter separately. At the end, I give some examples of data processing for the EQUATOR system.

SPEED INFLUENCE INVESTIGATION

Receiver motion

To analyze the influence of the receiver movement, let’s use the Faraday’s law written in the coordinate system associated with the receiver:

$$Emf_k = -\frac{d\Phi}{dt} = -\frac{d}{dt}\iint_{S_k}\mathbf{B}\cdot\mathbf{n}_k ds \approx -\frac{dB_k}{dt}\cdot S_k. \quad (2)$$

Here on the left side is the electromotive force in the k-th frame of the inductive sensor, Φ is the magnetic flux through this frame, calculated via the induction vector $\mathbf{B}$ and the frame area S_k, $\mathbf{n}_k$ is a unit vector orthogonal to the corresponding frame surface.

In case of a harmonic field $B_k = B_k^0(t)\cdot e^{i\omega t}$:

$$\frac{dB_k(t)}{dt} = i\omega B_k^0(t)\cdot e^{i\omega t}\left\{+\frac{dB_k^0(t)}{dt}\cdot e^{i\omega t}\right\}. \quad (3)$$

The second term in equation (3) is usually not considered. However, with the characteristics of EQUATOR we have:

- receiver oscilation frequency in flight $u \sim 2\pi$ 0.5 rad/s;
- amplitude of oscillations in flight $A \sim 0.02$ rad;
- field frequency $\omega \sim 2\pi\cdot 100$ rad/s;

and this term will be approximately 10^{-4} with respect to the first one ($\sim A\cdot u/\omega$). It means that if the sensitivity level is better than 10^{-4}, the receiver motion cannot be neglected. For this reason, in the EM4H system (Vovenko et al., 2013) it is not necessary to take into account the effect of the receiver motion, since the maximum sensitivity level for it is about this value. And for the EQUATOR it is necessary, as we can see in Figure 1. By introducing a correction into the measured signal proportional to the change in the corresponding harmonic amplitude

$$B_k^m(\omega) = -\frac{i}{\omega S_k} Emf_k(\omega) = B_k^0(\omega) - \frac{i}{\omega}\frac{dB_k^0(\omega)}{dt} \Rightarrow \\ B_k^c(\omega) = B_k^m(\omega) + \frac{i}{\omega}\frac{dB_k^m(\omega)}{dt} \quad (4)$$

the motion influence can be excluded (Figure 1). The error of the substitution dB_k^0/dt by dB_k^m/dt in (4) is about 10^{-8}.

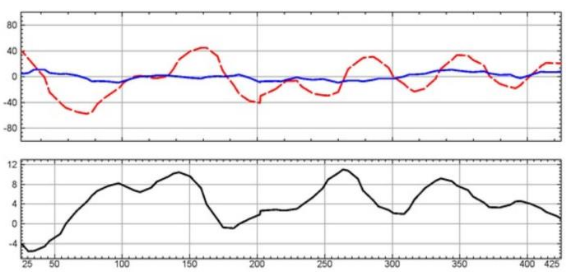

Figure 1. In the absence of the secondary field: measured (red) and corrected (blue) values in ppm as a function of time in samples (~ 10 Hz) for one of the EQUATOR's harmonics (230 Hz). The bottom chart is the receiver axis inclination in degrees.

Transmitter motion

Let's rewrite Maxwell's equations in the quasistatic approximation in the coordinate system associated with the Earth in the following form:

$$\nabla^2 \mathbf{B}^p + \mu_0 \sigma \frac{\partial \mathbf{B}^p}{\partial t} = 0. \quad (5)$$

$\mathbf{B}^p$ is the primary field, which can be expressed via dipole moment vector **M**:

$$\mathbf{B}^p = \frac{\mu_0}{4\pi|\mathbf{r}|^3}\left(3\frac{\mathbf{r}\mathbf{r}^T}{|\mathbf{r}|^2} - \mathbf{I}\right)\mathbf{M} = \Omega(\mathbf{r})\mathbf{M}\,. \quad (6)$$

r is the radius vector of the point with respect to the transmitter, **I** is the 3×3 identity matrix, $\mathbf{r}\mathbf{r}^T$ is the 3×3 matrix of the component wise products. Then the derivative of the primary field contains two terms:

$$\frac{\partial \mathbf{B}^p}{\partial t} = \underbrace{\frac{\partial \Omega(\mathbf{r}(t))}{\partial t}\mathbf{M}}_{(I)} + \underbrace{\Omega(\mathbf{r})\frac{\partial \mathbf{M}(t)}{\partial t}}_{(II)}. \quad (7)$$

The first term is

$$(I) = \frac{\partial \mathbf{B}^p(\mathbf{r})}{\partial \mathbf{r}} \cdot \frac{\partial \mathbf{r}(t)}{\partial t} = \nabla \mathbf{B}^p \mathbf{v}\,. \quad (8)$$

Thus, the velocity of the field source is a coefficient at the gradient of the primary field. The second term, in turn, has two components ($\boldsymbol{\upsilon}$ is the angular velocity of rotation of the transmitter frame, the value of |**M**| does not change):

$$(II) = (\boldsymbol{\upsilon} \times \mathbf{M})e^{i\omega t} + i\omega \mathbf{M} e^{i\omega t}. \quad (9)$$

Usually in AEM only the second part of the second term in equation (9) is taken into account. But in will be more accurate to write Maxwell's equations for the frequency ω in the following form:

$$\nabla^2 \mathbf{B}^p + i\omega\mu_0\sigma\mathbf{B}^p\{+\mu_0\sigma[\nabla\mathbf{B}^p\mathbf{v} + \Omega(\mathbf{r})(\boldsymbol{\upsilon}\times\mathbf{M})]\} = 0. \quad (10)$$

Using the characteristics of the EQUATOR system, namely:

- loop oscilation frequency in flight $u \sim 2\pi$ 0.1 rad/s;
- amplitude of oscillations in flight $A \sim 0.1$ rad;
- field frequency $\omega \sim 2\pi\cdot 100$ rad/s;
- flight speed $v = |\mathbf{v}| \sim 40$ m/s;
- loop height over ground $h \sim 40$ m;

the contribution of each term can be estimated. Thus, the contribution of the term, which includes the flight speed, is $v/(\omega\, h) \sim 10^{-3}$ with respect to the first, main term. Here it is taken into account that the dipole field near the surface is proportional to $1/h^3$, while the gradient is proportional to $1/h^4$. The contribution from the angular motion of the transmitting loop is $(A\, u)/\omega \sim 10^{-4}$.

Further, the last term of the equation (10) will not be taken into account. However, it should be kept in mind that at lower frequencies (~10 Hz) the angular motion of the transmitter loop can no longer be neglected.

The form of the influence of the speed-related term of the equation (10) is somewhat similar to the form of the field of a horizontal dipole directed in the direction of flight. However, the field under consideration decays faster with distance because it is related to the gradient. Figure 2 shows three components of the secondary magnetic field appeared due to the transmitter speed at the altitude of the transmitter – 40 m. They are presented as the parts of the stationary component. It can be seen that the horizontal component of the field is distorted by 0.2% on the transmitter axis (point(0,0)). The vertical component is distorted when moving in the flight direction (up to 0.25%).

Data analysis

I considered signals in the frequency domain in the range from 77 Hz to 14 kHz. I performed vertically constrained 1D inversion (Guillemoteau et al., 2011) with fixed layers having thickness of 4 meters and thicker. Over the shallow sea a large residual was obtained, about 5–10 units of the signal RMS, which was increasing with approaching to the coastline. To solve this problem, in addition to the resistivity the altimeter readings error was also estimated. Despite the fact that the solution obtained almost never differs from the measurements by more than 3 RMS, serious doubts arose, since the adequacy of the altimeter was checked many times. However, the height correction turned out to be up to 3 meters. Moreover, it correlates with the electrical conductivity of the medium.

To check if the transmitter speed is the cause of these distortions, the two lowest frequencies, 77 and 231 Hz, were excluded from processing. For higher frequencies the contribution of the speed-related part is negligible. As a result, in terms of height correction, the solution improved, but did not improve completely. As before, as the water depth decreases, the height correction began to increase, and its maximum value, as before, reached 3 meters.

Figure 3 shows the results of the inversion for the same set of frequencies (without two lowest), but not adjusting the height correction. The top graph shows the residuals calculated for

the two frequencies not involved in processing. It can be seen that the residuals for 77 Hz are much larger than for 231 Hz, which is consistent with the conclusions about the effect of speed. However, the magnitude of the residual (up to 4000 ppm) is almost two orders higher than the magnitude the estimate of the speed effect, which at a secondary field strength of about 20000 ppm in this case will be up to 0.25%, i.e. about 50 ppm. It should be noted that the measurement noise at this frequency is about 10 ppm. Thus, the speed of the transmitter at this stage of interpretation does not affect the result, while the layers grid obliviously does: height correction value never exceeds 4 meters – the thickness of the 1st layer.

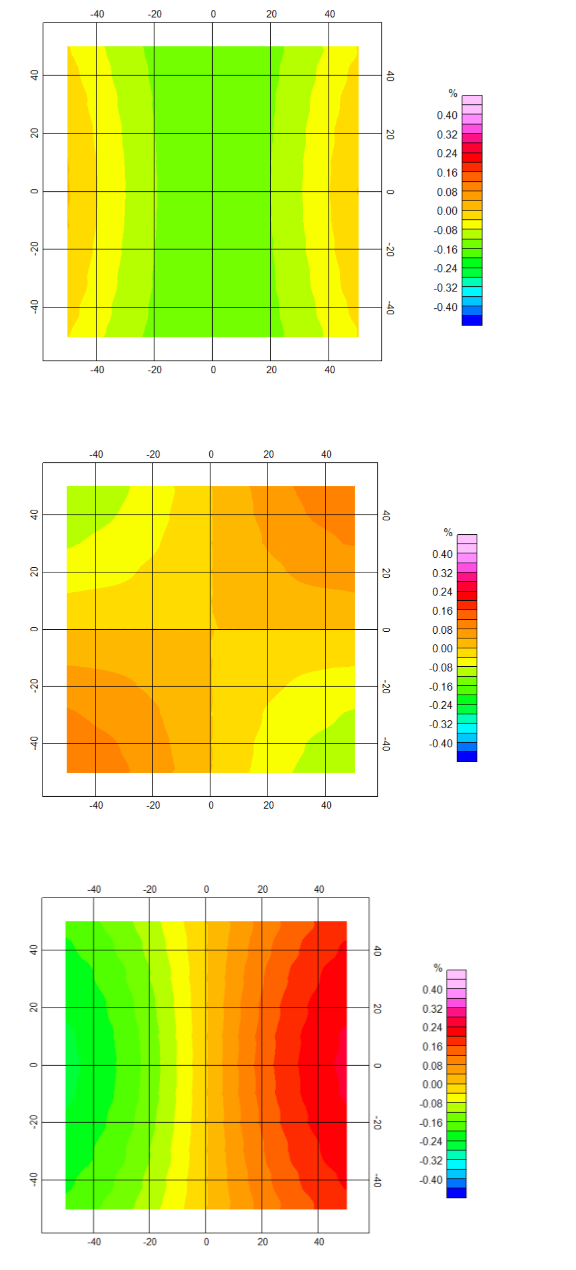

Figure 2. X, Y and Z components of the speed-induced field with respect to the stationary part.

CONCLUSIONS

With the existing level of sensitivity of the AEM equipment, it is necessary to take into account the movement of the receiver. The easiest way is to introduce a correction proportional to the change in the amplitude of the corresponding field harmonic.

Accounting for the transmitter speed for the EQUATOR system is not critical. The residuals obtained by 1D inversion of AEM data are explained by the grid spacing of the resistivity distribution model. Four meters thickness turned out to be too much in this specific case. The resulting height correction compensates for misadjustment of the boundary position between the conductor (sea water) and the more resistive base. It is for this reason that the height correction nowhere exceeds 4 meters.

Nevertheless, calculations show that when the frequency of the primary field signal decreases by an order (from 100 Hz to 10 Hz), it becomes necessary to take into account the aircraft speed.

ACKNOWLEDGMENTS

I want to thank my dear colleague, Dr. Pavel Aleksandrov, who has always emphasized the difference between mobile and stationary EM systems.

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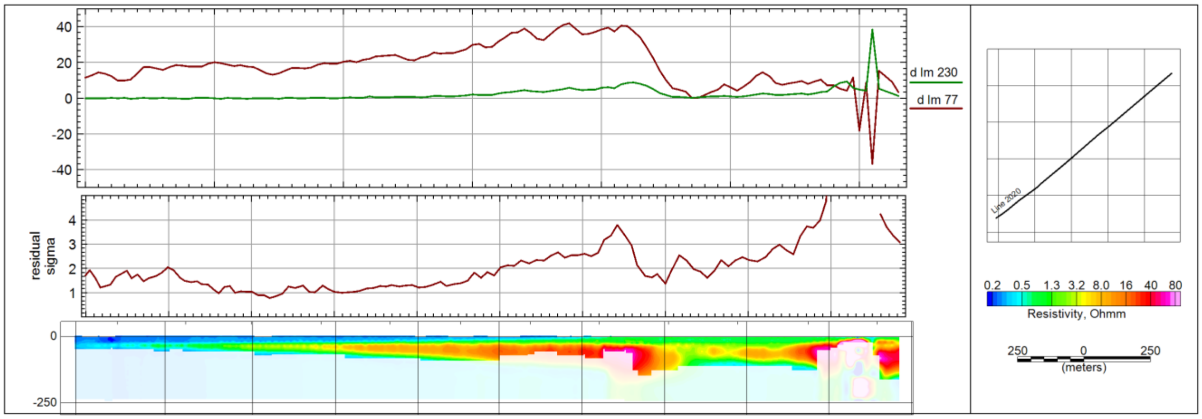

Figure 3. Inversion result in frequency domain without 77 and 230 Hz. Upper chart: d Im 77 – residual for 77 Hz, d Im 230 – residual for 230 Hz, both in ppm*100. Central chart: solution residual calculated for all other frequencies, normalized by the signal RMS. Bottom: the resistivity section.