

# Adaptive Algorithm of Quasi-Stationary Periodic Processes Measurements Control

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**Abstract**—The aim of the work was to create an effective method of control of linear measuring systems functioning in the conditions of the prevailing influence of distortions caused by variability in time of measuring conversion parameters. The method and the algorithm of control of measurements of the quasi-stationary periodic process spectrum in frequency-domain and form in time-domain are presented, consisting in the simultaneous separate observation of the parameters of the process under probe and the parameters of the measuring system with the subsequent introduction of corrections. The control of system parameters is carried out using the artificial stationary polyharmonic sample impact. The spectra of the main and sample processes are not intersected. A synthesizing method of the form in time-domain of the sample impact process is presented. The main limitations and the range of conditions for the possible application of the method are determined, its effectiveness is shown on the example of the experimental data obtained during the low-frequency inductive electrical prospecting system functioning in alternating magnetic field measuring mode.

*Keywords:* measurements control, conversion parameters control, adaptive correction, quasi-stationary periodic process

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## 1. INTRODUCTION

High-precision measurement of the parameters of various processes is a necessary and important element of the functioning of most control systems. The accuracy of measurements ultimately depends on the accuracy of the operation of various controllers, motion control systems and control of process parameters. A significant place in the measurement technique is occupied by the control of the parameters of cyclic quasi-stationary processes, which can be considered periodic over a certain interval of observation time. Such measurements often occur when the subject of the measurement is the nature of the reaction of an object to a known probing process. In this case, the sensors of the measuring system detect the impact of the process, also periodic, but generally unknown. The input is converted into an electrical signal and then digital information. Sensors involved in the measurement process, being complex according to the structure and principle of operation, often introduce significant distortions into the conversion results [1]. Conversion parameters are dependent on various kinds of external influences—primarily on thermal processes in the sensor itself and electronic units of the measuring system. The shape of the signal at the output of the measuring system becomes unpredictably different from that of the input. In the case of complex sensors, it may even happen that the distorting impact of the variability in measurement performance is much greater than that of noise and interference. For monitoring, control, tracking and many other tasks it is important to get an accurate picture of the process under study.

In this context, the management of the measurement process, which consists in monitoring the nature of measurement distortions and making adaptive corrective amendments to restore the true

parameters of the process under study, is an important and urgent task. Algorithmic aspects of its solution are the focus of this paper.

## 2. PROBLEM STATEMENT

Let the influence  $x$  on the sensitive elements of the sensor of the measuring system be considered stationary over the time interval  $(t-nT, t+nT)$  of  $2n$  signal periods ( $T = 2\pi/\omega$ ) and monochromatic with the frequency  $\omega$ :

$$x(t + \tau) = \operatorname{Re} (a(t)e^{j(\omega\tau + \varphi(t))}) = \operatorname{Re} (X(t, \omega)e^{j\omega\tau}),$$

where  $a$  is the amplitude,  $\varphi$  is the initial phase,  $\tau$  is the continuous time,  $t$  is the discrete (astronomical) time. With regard to the two time parameters, make the following comment. Since the behavior of the object under study and observation conditions on the stationarity interval are assumed to be invariable, it is convenient to present the results of measurements on the whole interval in a folded form, and to attribute a single value of this form to the average point of the stationarity interval, a discrete countdown of the system (astronomical) time scale  $t$  (as opposed to the continuous time scale  $\tau$ ).

A convenient form of this kind of representation is the value of the so-called complex signal amplitude of the corresponding frequency [2], in the above expression

$$X(t, \omega) = a(t, \omega)(\cos(\varphi(t, \omega)) + j \sin(\varphi(t, \omega))).$$

Input impact  $x$  is converted by the measuring system into an electrical output signal of a different shape, since any measuring conversion inevitably introduces various kinds of distortion. Within the stationary interval, the nature of the distortions can be considered constant and, assuming the linearity of the conversion, is represented in the form of the value of the complex conversion coefficient at the corresponding frequency assigned to the corresponding astronomical time, i.e.,  $W(t, \omega)$ . In addition, the impact of the noise additive  $s$  should be considered. With this in mind, the expression for the signal at the output of the measuring system shall be recorded in the form:

$$u(t + \tau) = \operatorname{Re} [W(t, \omega)X(t, \omega)e^{j\omega\tau}] + s(t + \tau) = \operatorname{Re} [U(t, \omega)e^{j\omega\tau}] + s(t + \tau). \quad (1)$$

The parameter  $U(t, \omega)$  expresses the value of the complex amplitude of the output signal distorted by the shape and variability of the measurement transform  $W(t, \omega)$ .

The signal  $u$  can be subjected to analog-to-digital conversion and computational processing, as a result of which the value of the complex amplitude  $U(t, \omega)$  can be determined. Note that in formula (1) none of the parameters  $X$  and  $W$  can be considered known, because the first is the subject of measurement and is a priori unknown, and the second, as noted above, depends on various external factors and, generally speaking, for different readings  $t$  can take different values.

To determine the complex amplitude, a synchronous detection (coherent accumulation) procedure is applied:

$$\tilde{U}(t, \omega) = \frac{1}{nT} \int_{t-nT}^{t+nT} f(\tau)e^{j\omega\tau} u(t + \tau) d\tau = U(t, \omega) + o(t, \omega). \quad (2)$$

The sign “tilde” on the left side of the expression reflects the fact that the value of the complex amplitude  $\tilde{U}(t, \omega)$  obtained from the calculations differs from the value  $U$  from the expression (1) by the error value introduced by the impact of noise:

$$o(t, \omega) = \frac{1}{nT} \int_{t-nT}^{t+nT} f(\tau)e^{j\omega\tau} s(t + \tau) d\tau.$$

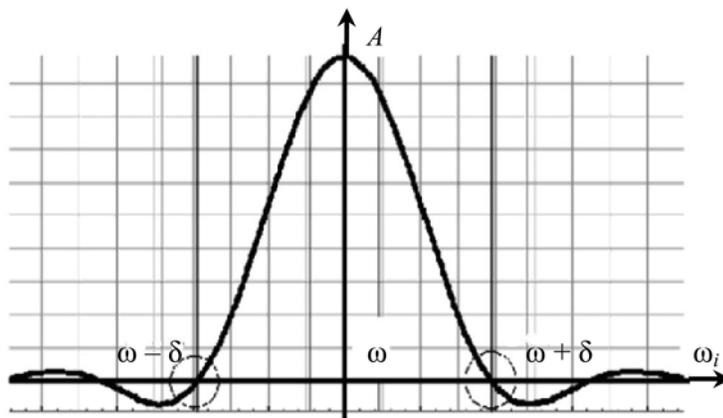


Fig. 1. Amplitude-frequency response of synchronous detector.

In the above expressions, the function  $f(\tau)$  defined at  $[-nT, +nT]$  is a weight function of coherent accumulation. The shape of the weight function significantly determines the quality (Q-factor) of detection, sensitivity to signals at different frequencies, both equal and different  $\omega_i$  from the detection frequency. The sensitivity of a synchronous detector to different frequencies  $\omega_i$  is defined as

$$A(\omega, \omega_i) = \frac{1}{nT} \int_{-nT}^{+nT} f(\tau) e^{j\omega\tau} \cos \omega_i \tau d\tau,$$

where  $A$  is the complex value of sensitivity of the synchronous detector to the impact of the signal at frequency  $\omega_i$  during detection at frequency  $\omega$ . The general shape of the amplitude component of this curve (amplitude-frequency response (AFR) of the detector) is shown in Fig. 1.

The dependence of the shape of the frequency response (FR) of the synchronous detector on the shape of the weight function is described in detail in book [3]. In the context of this work, it is important only that at the detection frequency the sensitivity takes the real value equal to one, and at the frequencies  $\omega \pm \delta$  (where  $\delta = m\pi/2nT$ ,  $m$  is integer) the sensitivity takes the value equal zero [4]. Thus, the presence of signals at these frequencies very close to the detection frequency  $\omega$  does not affect the result of the complex amplitude calculation. Since the value of  $\delta$  does not depend on the value of the frequency  $\omega$ , it is obvious that during synchronous detection at the frequencies  $\omega \pm \delta$ , the presence of a signal at the frequency of the main process  $\omega$  in its turn does not influence the detection result. This feature of synchronous detection is the basis for the method and algorithm of adaptive correction of measurement distortions proposed in this paper.

The measurement process control task in the context of the above is reduced to continuous monitoring of parameters of the measurement system conversion function directly in the process of observation of quasi-stationary input action with the subsequent introduction of adaptive correction, wherein the measurement conversion is assumed to be linear with a slowly changing in time

### 3. MEASUREMENT CONTROL, ADAPTIVE CORRECTION

Let us assume that the sensor of the measuring system, along with the periodic monochromatic influence on the stationarity interval with the frequency  $\omega$  (at unknown amplitude and phase values), is additionally affected by the artificially created known reference influence  $x_r$  of the form

$$x_r(t + \tau) = \text{Re} (X_L(\omega) e^{j(\omega - \delta)(t + \tau)}) + \text{Re} (X_R(\omega) e^{j(\omega + \delta)(t + \tau)}),$$

where  $X_L(\omega)$  and  $X_R(\omega)$  are the values of complex amplitudes of two components of biharmonic influence, i.e., influence at frequencies  $\omega - \delta$  and  $\omega + \delta$ , respectively. We will also assume that the values  $X_L(\omega)$  and  $X_R(\omega)$  are constant over the stationarity interval (the interval of coherent accumulation with the central point  $t$ ). Note that the last requirement is quite natural, since this effect is created artificially.

The impacts at the base frequency  $\omega$  and two associated additional reference frequencies  $\omega \pm \delta$ , adding up, form a frequency triplet at the input of the measuring system. After passing through various conversion stages taking into account the influence of imperfection of the measuring system, a signal of the following form

$$u_3(t + \tau) = \operatorname{Re} \left( W(t, \omega) e^{j\omega(t+\tau)} \right) + \operatorname{Re} \left( W(t, \omega - \delta) X_L(\omega) e^{j(\omega-\delta)(t+\tau)} \right) + \operatorname{Re} \left( W(t, \omega + \delta) X_R(\omega) e^{j(\omega+\delta)(t+\tau)} \right) + s(t + \tau)$$

will be observed at its output.

After receiving and amplifying the triplet signal can be synchronously detected on each of the three frequencies. Taking into account zero sensitivity of the synchronous detector to the frequencies, which are separated from the detection frequency by the frequency  $\delta$ , the values of three complex amplitudes will be obtained on the accumulation interval with the central value of time  $t$ :

$$\begin{aligned} \frac{1}{nT} \int_{t-nT}^{t+nT} f(t) u_3(t + \tau) e^{j\omega(t+\tau)} dt &= \tilde{U}(t, \omega) = W(t, \omega) X(t, \omega) + o(t, \omega), \\ \frac{1}{zT} \int_{t-zT}^{t+zT} f_1(t) u_3(t + \tau) e^{j(\omega-\delta)(t+\tau)} dt &= \tilde{U}_L(t, \omega) = W(t, \omega - \delta) X_L(\omega) + o(t, \omega - \delta), \\ \frac{1}{zT} \int_{t-zT}^{t+zT} f_1(t) u_3(t + \tau) e^{j(\omega+\delta)(t+\tau)} dt &= \tilde{U}_R(t, \omega) = W(t, \omega + \delta) X_R(\omega) + o(t, \omega + \delta). \end{aligned} \quad (4)$$

The above expressions show that the accumulation interval when detected at additional frequencies differs from the accumulation interval at the base frequency. It is important that an integer number of periods  $z > n$ , since as the accumulation time increases, the frequency band of the synchronous detection narrows, and if the measurement noise is considered white, then its power in the detection band decreases accordingly. This means that the detection error at the additional frequencies  $o(t\omega \pm \delta)$  becomes small enough to assume, in particular,

$$\tilde{U}_{R(L)}(t, \omega) = W(t, \omega \pm \delta) X_{R(L)}(\omega).$$

It is important to note that if the length of the stationary interval (accumulation interval) is sufficiently large with respect to the period  $T$ , then the frequency interval between the base and additional frequencies  $\delta = m\pi/nT$  is very small, and the frequency triplet band is narrow with respect to the detection frequency ( $\omega \gg 2\delta$ ). The frequency response in this band can be considered smooth, and in the first approximation—a linear function,—therefore it can be assumed that

$$\tilde{W}(t, \omega) = \frac{\tilde{U}_L(t, \omega)}{2X_L(\omega)} + \frac{\tilde{U}_R(t, \omega)}{2X_R(\omega)} = W(t, \omega) + o_W(t, \omega), \quad (5)$$

where according to formulas (3):  $o_W(t, \omega) = \left[ \frac{o(t, \omega - \delta)}{2X_L(\omega)} + \frac{o(t, \omega + \delta)}{2X_R(\omega)} \right]$ ;  $W$  is the true value of the conversion factor;  $\tilde{W}$  is a value calculated from the results of coherent accumulation at the frequencies of the reference impact (signal).

Expression (4) is remarkable in that the  $\tilde{W}(t, \omega)$  conversion coefficient turns out to be, albeit approximately, expressed in terms of quantities that are known a priori or obtained as a result of processing the output signal of the measuring system in the very process in which the shape of the input observed influence is recorded on the corresponding interval of coherent accumulation; simultaneously with the value of the complex amplitude  $\tilde{U}(t, \omega)$ , the value of the complex conversion factor for the measuring system at the same frequency is approximately determined. In the context of the problem of adaptive control of the measurement process, this means that the first part of the problem has been solved—the frequency characteristic of the system has been monitored.

Now it is possible to start implementing the adaptive correction. Knowing the value of  $\tilde{W}(t, \omega)$  and assuming  $o_W(t, \omega) \approx 0$ , it remains to correct the result of synchronous detection for the observed process, i.e., to find for it an approximate spectral form of the input action

$$\tilde{X}(t, \omega) = \frac{\tilde{U}(t, \omega)}{\tilde{W}(t, \omega)} = \frac{U(t, \omega)}{\tilde{W}(t, \omega)} + \left[ \frac{o(t, \omega)}{\tilde{W}(t, \omega)} \right]. \quad (6)$$

The conversion (5) completes the stage of correcting the adaptive algorithm for controlling the measurement of monochromatic process parameters.

With regard to the result obtained, it is important to evaluate the resulting error of the introduced correction due to the inevitable impact of noise. Assuming

$$W(t, \omega) \gg o_W(t, \omega)$$

and then expanding expression (5) in a series by the small parameter

$$\varepsilon = \frac{o_W(t, \omega)}{W(t, \omega)},$$

we receive, limited by linear approximation,

$$\begin{aligned} \tilde{X}(t, \omega) &= \frac{U(t, \omega)}{W(t, \omega)}(1 - \varepsilon) + \frac{o(t, \omega)}{W(t, \omega)}(1 - \varepsilon) \\ &\approx X(t, \omega) + \frac{o(t, \omega)}{W(t, \omega)} - \varepsilon X(t, \omega). \end{aligned} \quad (7)$$

The error value of the entered correction is represented by the last member in this expression, i.e.,  $\varepsilon X(t, \omega)$ . The component  $\frac{o(t, \omega)}{W(t, \omega)}$  represents the synchronous detection error, referred to the input of the measuring system. This value according to expression (6) is independent of the correction.

The expected mean square deviation of complex amplitude calculation errors according to [5] is approximately

$$o(t, \omega \pm \delta) \sim \sqrt{\sigma^2/4z},$$

where  $\sigma$  is the mean square deviation of noise directly at the output of the measuring system, and  $z$  is the number of readings in the synchronous detection sample. Since the reference impact can be confidently assumed to be stationary, a significant accumulation interval can be used for the reference frequencies. The introduction of a correction does not lead to an increase in the error of determining the complex amplitude  $X$  of the base process. Thus, we will further assume

$$\tilde{W}(t, \omega) = W(t, \omega) \quad \text{as well as} \quad \tilde{X}(t, \omega) = X(t, \omega).$$

It is important to note here that in practice, far from all experiments, the distorting impact of noise is decisive. Very often, the main factor is precisely the unpredictability and variability of the parameters of the measuring conversion.

#### 4. ADAPTIVE CORRECTION IN A COMPLEX TIME FORM OF THE INVESTIGATED PROCESS

For signal  $u(\tau)$  with period  $T$ ,  $u(\tau) = u(\tau + T)$ .

Therefore, in the process of calculating the complex amplitude for the frequency  $\omega$  with coherent accumulation in formula (2), it is possible to change the summation order in this way:

$$\tilde{U}(t, \omega) = \frac{1}{nT} \int_0^T \left[ \sum_{i=-n}^{n-1} (f(iT + \tau)u(t + iT + \tau)) \right] e^{j\omega\tau} d\tau = \frac{1}{nT} \int_0^T \nu(t, \tau) e^{j\omega\tau} d\tau \quad (8)$$

where  $\nu(t\tau)$  is a function defined on the segment  $[0, T]$ , which represents the so-called time accumulation data array—a short graph of the average waveform for one period. Thus, when calculating the value of the complex amplitude, the signal with the weight function  $f(\tau)$  on the segment corresponding to the period  $T$  is first temporarily accumulated, and upon its completion, synchronous detection is carried out at the interval  $(0, T)$  at the frequency  $\omega$  at the unit weight function.

Note that the presence at the output of the measuring system of signals with frequencies  $\omega \pm \delta$  does not change the shape of the time accumulation function, since

$$e^{j(\omega+\delta)(t+\tau)} = e^{j\delta(t+\tau)} e^{j\omega(t+\tau)}, \quad e^{j\delta(t \pm nT)} = -e^{j\delta t},$$

and the time period  $(-nT, nT)$  corresponds to exactly one frequency period  $\delta$ .

It is also important to note that expression (7) is valid for all values of  $\omega$ , divisible by  $2\pi/T$  (the function periodic with period  $T$  is also periodic with period  $rT$ , where  $r = 1, 2, \dots$ ).

Thus, the biharmonic reference signal is “invisible” both with synchronous detection at the frequency  $\omega$  and with time accumulation over a segment of duration  $rT$ .

To extract the time form of the signal, which is a conversion of reference impact only, we “modulate” the total triplet signal with frequency cosine  $\delta$  (this operation is purely computational and is performed on the data by the computer).

Expression

$$\begin{aligned} \tilde{u}_r(t, \tau) = & \operatorname{Re} \left[ \tilde{U}(t, \omega) e^{i\omega(t+\tau)} e^{j\delta(t+\tau)} \right] + \operatorname{Re} \left[ \tilde{U}_L(t, \omega) e^{i(\omega-\delta)(t+\tau)} \right] \\ & + \operatorname{Re} \left[ \tilde{U}_R(t) e^{i(\omega+\delta)t} \right] (e^{j\delta(t+\tau)} + e^{-j\delta(t+\tau)}) / 2 \end{aligned}$$

by identical transformations can be reduced to the following form:

$$\begin{aligned} \tilde{u}_r(t, \tau) = & \frac{1}{2} \left[ \operatorname{Re} \left( \tilde{U}(t, \omega) e^{j(\omega-\delta)(t+\tau)} \right) + \operatorname{Re} \left( \tilde{U}(t, \omega) e^{j(\omega+\delta)(t+\tau)} \right) \right. \\ & + \operatorname{Re} \left( \tilde{U}_L(t, \omega) e^{j(\omega-2\delta)(t+\tau)} \right) + \operatorname{Re} \left( \tilde{U}_R(t, \omega) e^{j(\omega+\delta)(t+\tau)} \right) \\ & \left. + \operatorname{Re} \left( \tilde{U}_L(t, \omega) + \tilde{U}_R(t, \omega) \right) e^{j\omega(t+\tau)} \right]. \end{aligned} \quad (9)$$

If now this synthetic signal is subjected to synchronous detection at the frequency  $\omega$ , then it turns out that according to formula (8) only the components of the reference signal are “visible” (the synchronous detector is insensitive to both the frequencies  $\omega \pm \delta$  and the frequencies  $\omega \pm 2\delta$ ). Signals

caused by reference action are combined into a single harmonic signal at the base frequency. Thus, by synchronous detection of the  $\tilde{u}_r(t, \tau)$  signal at the frequency  $\omega$  according to (3), the following value will be obtained

$$\begin{aligned}\tilde{U}_r(t, \omega) &= \frac{1}{nT} \int_0^T \left( \sum_{i=-n}^{n-1} f(iT + \tau) \tilde{u}_r(t + iT + \tau) \right) e^{j\omega\tau} d\tau = \frac{1}{nT} \int_0^T v_r(t, \tau) e^{j\omega\tau} d\tau \\ &= \frac{1}{2} (W(t, (\omega - \delta)) X_L(\omega) + W(t, \omega + \delta) X_R(\omega)) + o'(t, \omega).\end{aligned}\quad (10)$$

Taking into account that the measurement noise in the detector's sensitivity band is assumed to be white, the statistical parameters of the average values

$$\overline{o'(t, \omega)} = \overline{o(t, \omega)}$$

are retained [4]. If it is known that

$$X_L(\omega) = X_R(\omega) = X_0(\omega),$$

then according to the formula (9),

$$\tilde{W}(t, \omega) \approx \frac{W(t, \omega - \delta) + W(t, \omega + \delta)}{2} \approx \frac{\tilde{U}_r(t, \omega)}{2X_0(\omega)}.\quad (11)$$

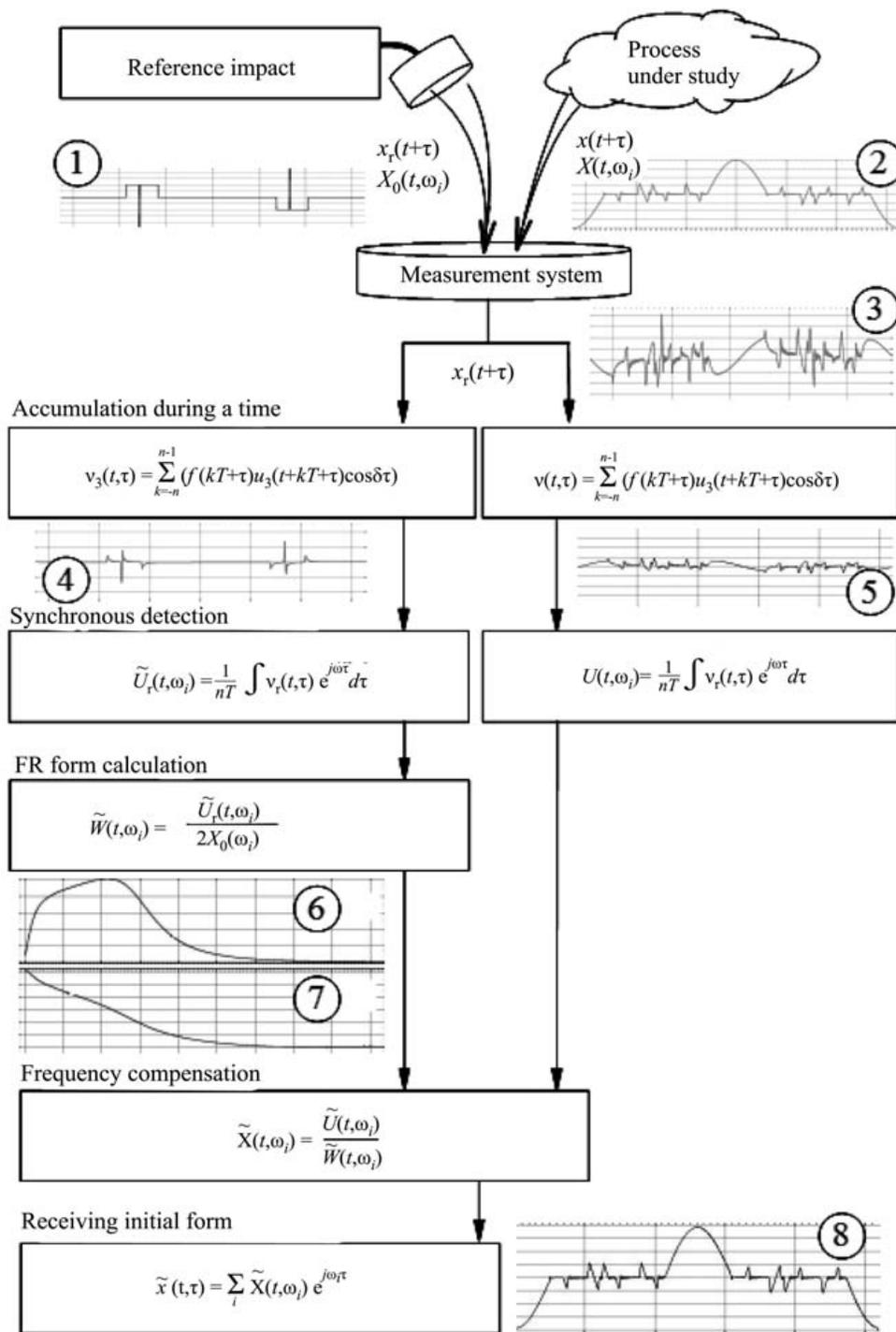
Thus, with direct time accumulation on the period  $T$  it is possible not only to allocate a monochromatic signal at the base frequency  $\omega$ , but also to determine the conversion factor for this frequency.

Let now the input action investigated in the experiment is polyharmonic, that is, it is represented by a set of harmonics that are multiples of the repetition frequency of the periodic signal  $\omega_0$ , i.e., by the spectrum  $X(\omega_i)$ , where  $\omega_i = i\omega_0$ ,  $i = 1, 2, \dots$ . Note also that the harmonic set is always finite in one way or another, it is limited by the frequency range of the sensors of the measuring system. If the sensor of the measuring system, along with the base impact under study, is subjected to an additional reference impact, represented by a set of pairs of harmonic processes, then

$$\begin{aligned}x(t + \tau) &= \sum_i \operatorname{Re} (X(t, i\omega_0) e^{j(i\omega_0)\tau}) + \sum_i \operatorname{Re} (X_L(t, i\omega_0) e^{j(i\omega_0 - \delta)\tau}) \\ &\quad + \sum_i \operatorname{Re} (X_R(t, i\omega_0) e^{j(i\omega_0 + \delta)\tau}).\end{aligned}$$

Thus, the same triplet structure is applied for each of the frequencies. According to formula (8), if  $4\delta < \omega_0$ , the spectra of the base and reference impact do not intersect. If the above conditions are met, then all the calculations presented above are valid for each triplet with a central frequency  $\omega_i$ , and, consequently, for the entire complex periodic signal. This means that using the above algorithm, the measurement distortions for each of the spectrum frequencies can be corrected not only for monochromatic but also for periodic signals of complex shape.

The principle of time accumulation is especially effective in adaptive control of the process of measuring periodic processes of a complex form. For all frequencies of the  $X(\omega_i)$  spectrum, a single time accumulation array  $\nu(t, \tau)$  of the base signal form and a single array  $\nu_r(t, \tau)$  of the reference impact form can be used (if only  $X_L(\omega_i) = X_R(\omega_i) = X_0(\omega_i)$ ).



**Fig. 2.** Calculation scheme of adaptive measurement control algorithm: 1—reference impact (before modulation with frequency  $\delta$ ); 2—input impact of the investigated process; 3—signal at the output of the measuring system (reference and base processes in total); 4—reference signal after time separation and accumulation; 5—signal of investigated process after separation and accumulation; 6, 7—amplitude-frequency response (AFR) and phase frequency response (PFR) of the measuring system, calculated from the results of computational processing; 8—correction result, restored form of impact.

After time accumulation, synchronous detection can be performed on each of the arrays  $\nu(t, \tau)$  and  $\nu_r(t, \tau)$  at all actual frequencies of the set  $X(\omega_i)$ . As a result of this operation, two sets of complex amplitude values—two spectra:  $\tilde{U}(t, \omega_i)$  and  $\tilde{U}_r(t, \omega_i)$ —will be obtained according to formulas (7) and (9). By the formula (10), for each of the frequencies  $X(\omega_i)$ , the value  $\tilde{W}(t, \omega_i)$  (adaptive control algorithm monitoring stage) can be calculated, and then frequency domain correction is performed

$$X(t, \omega_i) \approx \tilde{X}(t, \omega_i) = \frac{\tilde{U}(t, \omega_i)}{\tilde{W}(t, \omega_i)},$$

as well as the shape of the studied input impact is corrected

$$x(t + \tau) \approx \tilde{x}(t + \tau) = \sum_i \tilde{X}(t, \omega_i) e^{j\omega_i(t+\tau)}.$$

Thus, the problem of adaptive control of measurements of a quasi-stationary periodic process can be considered solved. Here it is important to focus on some details of the possible implementation of the method.

The given algorithm of adaptive control allows to apply a simple method of synthesis of the form of the reference impact process. Taking into account that

$$\cos \alpha \cos \beta = 1/2[\cos(\alpha + \beta) + \cos(\alpha - \beta)],$$

it is enough to create a reference impact in the form of a periodic process with a period corresponding to the frequency  $\omega_0$ , and then modulate it at the hardware level with a sinusoidal frequency signal  $\delta$ . The spectrum of this impact for each of the  $\omega_i$  frequencies will contain a pair of harmonic components at  $\omega_i \pm \delta$  frequencies. The results of time accumulation for the base signal are not affected by this additional impact. The amplitudes of the components of each pair are the same, but generally speaking, they may differ for different  $\omega_i$  frequencies. If we set  $X_0(\omega_i) = \text{const}$  for all frequencies (this case corresponds to the reference signal in the form of infinitely narrow bipolar impulses), then the calculated signal  $\tilde{u}_r(t + \tau)$  directly describes the impulse response of the system up to a complex factor, and the spectrum  $\tilde{U}_r(t, \omega_i)$  describes the frequency response. Unfortunately, this type of input impact is difficult to implement at the hardware level.

The scheme of algorithmic implementation of the method of adaptive control of measurements of quasi-stationary periodic processes is given in Fig. 2, where the principle of formation of the reference impact and the sequence of calculations are schematically shown. The calculation progress and intermediate results are illustrated by graphs obtained in the processing of real physical experiment data [6].

Comparison of the forms of input impact and the correction result clearly demonstrates the effectiveness of the proposed method and algorithm.

## 5. CONCLUSIONS

The efficiency of the proposed method and the corresponding algorithm largely depends on how fully the measurement conditions are observed, among which we will single out the following.

- The value of the test signal period shall be accurately determined by the clock of the measuring system. If the process under investigation is the result of a reaction to a known probing impact, that is, the experiment as a whole is controlled by a single computer system, this requirement is met automatically.
- At the hardware level, the reference impact shall be as consistent as possible with the base observed process. Moreover, both the main observed and reference impacts must be applied simultaneously as a linear superposition to the same sensor of the measurement system. For

example, for adaptive correction in variable magnetic field measurements, the reference impact must be excited by a source of variable magnetic field—a dipole or dipole system stable relative to the sensing element. The accuracy of the correction in this case will depend on how fully it is possible to eliminate distortions in the transmission of the reference impact from the source to the sensor of the measuring system.

- The correction is not applicable to the constant component of the observed periodic signal. This fundamental limitation is not always essential. During the preparation phase of the experiment, either the impact itself or the operation of the measuring system can be organized so that this condition is met [3].
- According to the stationarity condition, not only all measured impacts, but also the frequency response of the measuring system must remain sufficiently constant during the accumulation interval.
- The base measurable impact is usually, although slow, but still variable in time. The purpose of measurements is usually to determine these changes. However, the variability of the periodic signal parameters at the input of the synchronous detector inevitably leads to the appearance of additional components in the spectrum. For the adaptive correction algorithm to work, it is necessary that these changes do not significantly affect the signal detection at additional frequencies. Hence, it turns out that, on the one hand, to ensure the accuracy of determining the parameters of the measuring system, the additional frequencies should be located as close as possible to the frequency of the measured signal, and on the other hand, sufficiently far away to exclude the influence of modulation of the base signal on the detection result at additional frequencies. This compromise can be resolved to a large extent by using a reference signal in the detector with a substantially longer (in comparison with the base signal detector) coherent accumulation time interval. In this case, the detection band is narrowed, and by its nature even the artificially created reference signal distorted by the measuring system changes in time slightly and slowly.
- At a relatively narrow interval  $\omega - \delta \leq \omega \leq \omega + \delta$ , the frequency response of the receiving system can be considered a linear function of frequency only under the condition  $\omega \gg \delta$ . At substantially low frequencies of the investigated process, commensurate with the frequency  $\delta$ , the adaptive algorithm is ineffective.

In general, these limitations meet the classical conditions for solving many problems: remote electromagnetic sensing of conductive media [7], radio and acoustic locations [8], etc.

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